Indian Institute of Technology Ropar
Department of Mathematics
MA101 - Calculus
First Semester of Academic Year 2025-26

Tutorial Sheet - 9

1. Using the limit definition of partial derivative, find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at the point $(1,2)$ for $f(x,y) = 1 - x + y - 3x^2y$.

2. Find the partial derivative of the functions with respect to each variable:
   (a) $f(\rho, \phi, \theta) = \rho \sin(\phi)$
   (b) $f(x,y) = \displaystyle\sum_{n=0}^{\infty} (xy)^n$ where $|xy| < 1$.

3. Find the value of $\dfrac{\partial z}{\partial x}$ at the point $(1,1,1)$, if the equation $xy + z^3x - 2yz = 0$ holds, where z defines as a function of the independent variables x and y and the partial derivatives exists.

4. Find $\dfrac{\partial x}{\partial u}$ and $\dfrac{\partial y}{\partial v}$, if the equations $u = x^2 - y^2$ and $v = x^2 - y$ where x and y defines as functions of the independent variables u and v, and the partial derivative exist. Then let $s = x^2 + y^2$ and find $\dfrac{\partial s}{\partial u}$.

5. Express $\dfrac{\partial w}{\partial t}$ as a function of t, both by using the chain rule and by expressing w in terms of t before differentiating, where $w = \ln(x^2 + y^2 + z^2)$, $x = \cos t$, $y = \sin t$, $z = 4\sqrt{t}$.

6. Express $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ as a function of u and v, both by using the chain rule and by expressing z in terms of u and v before differentiating where $z = 4e^x \ln y$, $x = \ln(u \cos v)$ and $y = u \sin v$.

7. Suppose that the partial derivatives of a function $f(x,y,z)$ at the point on the helix $x = \cos t$, $y = \sin t$, $z = t$ are $f_x = \cos t$, $f_y = \sin t$, $f_z = t^2 + t - 2$. At what points on the curve, if any, can f take on extreme values?

8. Find the derivative of the functions at P_0 in the direction of $\vec{A}$ using definition and also using gradient,
   (a) $f(x,y) = 2xy - 3y^2$, $P_0 = (5,5)$, $\vec{A} = 4\hat{i} + 3\hat{j}$.
   (b) $g(x,y,z) = 3e^x \cos(yz)$, $P_0 = (0,0,0)$, $\vec{A} = 2\hat{i} + \hat{j} - 2\hat{k}$.

9. Find the directions in which the function $f(x,y,z) = \dfrac{x}{y} - yz$ increase and decrease rapidly at point $P_0 = (4,1,1)$. Then find the derivatives of the function in these directions.

10. In what direction, the derivative of $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ at $P(3,2)$ equal to zero?

11. In the following parts, find an equation for the plane, tangent to the level surface $f(x, y, z) = c$ at P_0. (a)$x^2 - y - 5z = 0,\ P_0 : (2, -1, 1)$ (b) $x^2 + y^2 + z = 4,\ P_0 : (1, 1, 2)$

12. If $\left.\frac{df}{ds}\right|_{\vec{u},P_0}$ exists, then $\left.\frac{df}{ds}\right|_{c\vec{u},P_0}$ exists and also show that $\left.\frac{df}{ds}\right|_{c\vec{u},P_0} = c\left.\frac{df}{ds}\right|_{\vec{u},P_0}$, where c is any non-zero real number and f defines the function of two independent variables x and y.

13. (a)Find the total derivative of $f(x, y) = \sin xy + x^2y$ at the point $(1, \pi)$.
(b)Find the total derivative of $f(x, y) = (e^xy + x^2y, x^2 + y^2, \frac{x}{y})$ at the point $(1, -1)$.
(c)Find the total derivative of $g\,o\,f$ at the point $(-3, 1)$ where $f(x, y) = (x^2+y^2, xy)$ and $g(x, y) = (4xy, x - y, 3x^2 + 2y^2)$.

14. What is the largest value that the directional derivative of $f(x, y, z) = xyz$ can have at the point (1,1,1)?

15. Find a vector indicating the direction of most rapid increase of $f(x, y)$ at the given point. Then find the rate of change in that direction.
(a) $f(x, y) = e^y \sin x$ at $\left(\frac{5\pi}{6}, 0\right)$
(b) $f(x, y) = x^2y - \frac{2}{xy}$ at $(1, 1)$

16. The temperature at (x, y, z) of a ball centered at the origin is $T = 100e^{-(x^2+y^2+z^2)}$. Show that the direction of the greatest decrease in temperature is always a vector pointing away from the origin.

17. For the following functions, check whether $\left.\frac{\partial^2 f}{\partial x \partial y}\right|_{(0,0)} = \left.\frac{\partial^2 f}{\partial y \partial x}\right|_{(0,0)}$ holds or not. If yes, show it. If not, give reasons.

(a)$f(x, y) = xy + \frac{e^y}{y^2 + 1}$

(b)$f(x, y) = \begin{cases} \frac{xy\,(y^2 - x^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

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