



# Indian Institute of Technology Ropar

## Department of Mathematics

### MA101 - Calculus

#### First Semester of Academic Year 2025-26

##### Tutorial Sheet - 8

1. For the given functions  $f(x, y)$ , (a) find the function's domain, (b) find the function's range, (c) find the function's level curves, (d) find the boundary of the domain (e) Determine whether the domain is an open region, a closed region, or neither, and (f) decide whether the domain is bounded or unbounded:

$$(a) f(x, y) = \frac{y}{x^2} \quad (b) f(x, y) = \sqrt{9 - x^2 - y^2} \quad (c) f(x, y) = \sin^{-1}(y - x)$$

2. Find an equation for the level surface of the function  $f(x, y, z) = \ln(x^2 + y + z^2)$  passing through  $(-1, 2, 1)$ .

3. By considering different paths of approach, show that functions  $f(x, y)$  have no limit as  $(x, y) \rightarrow (0, 0)$ :

$$(a) f(x, y) = \frac{2xy}{x^2 + y^2} \quad (b) f(x, y) = \frac{xy}{|xy|} \quad (c) f(x, y) = -\frac{x}{\sqrt{x^2 + y^2}}$$

4. Does knowing that  $1 - \frac{x^2 y^2}{3} < \frac{\tan^{-1}(xy)}{xy} < 1$ , can you tell anything about  $\lim_{(x,y) \rightarrow (0,0)} \frac{\tan^{-1}(xy)}{xy}$ ?

5. Does knowing that  $2|xy| - \frac{x^2 y^2}{6} < 4 - 4 \cos(\sqrt{|xy|}) < 2|xy|$ , can you tell anything about  $\lim_{(x,y) \rightarrow (0,0)} \frac{4 - 4 \cos(\sqrt{|xy|})}{|xy|}$ ?

6. Find the limit if it exists or show that limit does not exist:

$$(a) \lim_{(x,y) \rightarrow (0,0)} \frac{y^2}{x^2 + y^2}$$

$$(b) \lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 y}{x^2 + y^2}$$

$$(c) \lim_{(x,y,z) \rightarrow (\pi, 0, 1/3)} e^{y^2} \tan(xz)$$

7. Using  $\epsilon - \delta$  definition, prove that following functions  $f(x, y)$  are continuous at a point  $(0, 0)$ :

$$(a) f(x, y) = x^2 + y^2 \quad (b) f(x, y) = \frac{x + y}{x^2 + 1} \quad (c) f(x, y) = \tan^2 x + \tan^2 y$$

8. Where the function  $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$  is continuous?

9. Determine the set of points at which function is continuous:

$$(a) f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

$$(b) f(x, y) = \begin{cases} \frac{3x^2y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

10. Prove that there is a number  $\delta > 0$  such that if  $x^2 + y^2 < \delta^2$ , then  $|x^2 + y^2 + 3xy + 180xy^5| < \frac{1}{10000}$ .

11. Can  $\frac{\sin(x + y)}{x + y}$  be made continuous by suitable defining it at  $(0, 0)$ ?

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