Indian Institute of Technology Ropar
Department of Mathematics
MA101 - Calculus
First Semester of Academic Year 2025-26

Tutorial Sheet - 7

1. Check the convergence of the improper integral $\int_0^3 \frac{dx}{x-1}$.

2. For what values of p does the integral $\int_1^\infty \frac{dx}{x^p}$ converges and what is its value?

3. Let $a < b$ and $p < 1$. Does the improper integral $\int_a^b \frac{dx}{(b-x)^p}$ converge?

4. Evaluate the improper integral $\int_2^\infty \frac{x+3}{(x-1)(x^2+1)}dx$.

5. Check the convergence of the following improper integrals:

   (a) $\int_1^\infty \frac{dx}{1+e^x}$ (b) $\int_1^\infty \frac{\cos x}{x^p}dx$ for $p > 0$.

6. Use integration, direct comparison test or limit comparison test to test the integrals for convergence:

   (a) $\int_0^\pi \frac{dt}{\sqrt{t}+\sin t}$ (b) $\int_0^{\ln 2} x^{-2}e^{\frac{-1}{x}}dx$ (c) $\int_1^\infty \frac{e^x}{x}dx$

7. Calculate the arc length:
   (a) $24xy = y^4 + 48$ from the point $(\frac{4}{3}, 2)$ to $(\frac{11}{4}, 4)$.
   (b) $x = 8t^{\frac{3}{2}}, y = 3 + (8-t)^{\frac{3}{2}}$ where $0 \le t \le 4$.

8. Calculate the area of the surfaces generated by revolving the arcs
   (a) $y = x^3$ from $x = 1$ to $x = 2$ about the x-axis.
   (b) $y = x + 2$ from $y = 2$ to $y = 5$ about the line $y = 4$.

9. An electric cable is hanging between two poles that are 200 meters apart and cable is in the shape of the graph of the function $y = 75(e^{\frac{x}{150}} + e^{\frac{-x}{150}})$. Find the length of the cable.

10. Let $(3\cos t, 4\sin t)$ represents a curve in $\mathbb{R}^2$ for $0 \le t \le 2\pi$. Find the approximate arc length of the curve when $\frac{\pi}{4} \le t \le \frac{3\pi}{4}$ so that the magnitude of error is less than 0.1

11. Use integration, direct comparison test or limit comparison test to test the integrals for convergence:

    (a) $\int_4^\infty \frac{dx}{x^2-2x}$ (b) $\int_1^\infty \frac{dx}{\sqrt{x^6+1}}$ (c) $\int_3^\infty \frac{dx}{2+\cos x+\ln x}$

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