Indian Institute of Technology Ropar
Department of Mathematics
MA101 - Calculus
First Semester of Academic Year 2025-26

Tutorial Sheet - 3

1. Discuss the convergence of the following series:

   (a) $\displaystyle\sum_{n=1}^{\infty}(\sqrt{n^2+1}-n)$.

   (b) $1+\dfrac{2^{1/100}}{2}+\dfrac{3^{1/100}}{3}+\dfrac{4^{1/100}}{4}+\cdots+\infty$.

2. Check whether the following series are convergent or divergent:

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{1}{n^2}\right)$.

   (b) $\displaystyle\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{1}{n}\right)$.

3. Discuss the convergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{x^n+a^n}$, $x, a > 0$.

4. If $\displaystyle\sum_{n=1}^{\infty}u_n$ is a convergent series of non-negative terms such that $u_n \neq 1$, then prove that $\displaystyle\sum_{n=1}^{\infty}u_n^2$ and $\displaystyle\sum_{n=1}^{\infty}\frac{u_n}{1-u_n}$ are both convergent.

5. Using the integral test, discuss the convergence of the following:

   (a) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\log(n))^p}$, $p > 0$.

   (b) $\displaystyle\sum_{n=1}^{\infty}ne^{-n^2}$.

6. Using D'Alembert's ratio test, prove the following series are convergent:

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{n^n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n!}$.

7. Determine if each of the following series are absolutely convergent, convergent or divergent:

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}, p > 0.$

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n^2}$

   (c) $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}.$

8. Using Cauchy's root test, discuss the convergence of the following series:

   (a) $\sum_{n=1}^{\infty} e^{-\sqrt{n}} r^n,\ r \geq 0.$

   (b) $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \ldots + \infty.$

9. For what values of x, does the following power series (a)converges conditionally and (b)converges absolutely. Also find the interval of convergence:

   (a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}.$

   (b) $\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2}.$

10. Find the interval of convergence of the following series and within that interval find the sum of the series as a function of x.

$$\sum_{n=1}^{\infty} \frac{(x+1)^{2n}}{4n}.$$

11. Show that if two power series $\sum_{n=1}^{\infty} a_n x^n$ and $\sum_{n=1}^{\infty} b_n x^n$ are convergent and equal for all values of x in an open interval $(-c, c)$, then $a_n = b_n$ for all n.

12. Find the Taylor's series generated by f at $x = a$, where $f(x) = 2^x$ and $a = 1$.

13. Find the Maclaurin series for the function $f(x) = \frac{1}{1-x}$.

14. Find the third degree Taylor polynomial $T_3(x)$ for $f(x) = \sin x$ centered at $x = \frac{\pi}{6}$ to estimate $\sin(35^o)$ correct to five decimal places.

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