Indian Institute of Technology Ropar
Department of Mathematics
MA101 - Calculus
First Semester of Academic Year 2025-26

Tutorial Sheet - 12

* Triple integrals and its application, change of variables, line integral, vector integration.

1. Evaluate $\int\int\int x^2yz\,dx\,dy\,dz$ throughout the volume bounded by the planes $x=0$, $y=0$, $z=0$, $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

2. Find the volume of the region bounded by the surface $y=x^2$, $x=y^2$ and the planes $z=0$, $z=3$.

3. Show that the volume of the wedge intercepted between the cylinder $x^2+y^2=2ax$ and the planes $z=mx$, $z=nx$ is $\pi(m-n)a^3$.

4. Evaluate $\int\int\int xz\,dV$ over E which is above $x^2+y^2+z^2=4$, inside the cone (pointing downward) that makes an angle of $\frac{\pi}{3}$ with the negative z-axis and has $x\le 0$.

5. Evaluate $\int\int\int y\,dV$ over E which is the region that lies below the plane $z=x+2$ above the xy- plane and between the cylinders $x^2+y^2=1$ and $x^2+y^2=4$.

6. Integrate
$$\int_0^3\int_0^4\int_{x=y/2}^{x=(y/2)+1}\left(\frac{2x-y}{2}+\frac{z}{3}\right)dx\,dy\,dz$$
by applying the transformation $u=\frac{2x-y}{2}$, $v=\frac{y}{2}$, $w=\frac{z}{3}$ and integrating over an appropriate region in the uvw-plane.

7. Evaluate $\int\int(x+y)\,dA$ over R which is the trapezoidal region with vertices given by $(0,0)$, $(5,0)$, $(5/2,5/2)$ and $(5/2,-5/2)$ using the transformation $x=2u+3v$ and $y=2u-3v$.

8. (a) Evaluate $\int_C(xy+y+z)ds$ along the curve $r(t)=2t\vec{i}+t\vec{j}+(2-2t)\vec{k}$, $0\le t\le 1$.
(b) Integrate $f(x,y,z)=x+\sqrt{y}-z^2$ over the path from $(0,0,0)$ to $(1,1,1)$ given by
$$C_1: r(t)=t\vec{i}+t^2\vec{j},\ 0\le t\le 1$$
$$C_2: r(t)=\vec{i}+\vec{j}+t\vec{k},\ 0\le t\le 1$$

9. Find the area of the windind wall standing orthogonal on the curve $2x+3y=6$, $0\le x\le 6$ and beneath the curve on the surface $f(x,y)=x+\sqrt{y}$.

10. Find the gradient fields of the function $f(x,y,z) = \ln\sqrt{x^2+y^2+z^2}$.

11. Find the line integral of $F = \sqrt{z}\vec{i} - 2x\vec{j} + \sqrt{y}\vec{k}$ over the path $C_1 \cup C_2$ consisting the line segment from $(0,0,0)$ to $(1,1,0)$ followed by the segment from $(1,1,0)$ to $(1,1,1)$.

12. Find the work done by F over the curve in the direction of increasing t.

$$F = 6z\vec{i} + y^2\vec{j} + 12x\vec{k},$$

$$r(t) = \sin t\vec{i} + \cos t\vec{j} + (t/6)\vec{k},\ 0 \le t \le 2\pi$$

13. Find the work done by the force $F = xy\vec{i} + (y-x)\vec{j}$ over the straight line from $(1,1)$ to $(2,3)$.

14. Find the circulation and flux of the fields $F_1 = x\vec{i} + y\vec{j}$ and $F_2 = -y\vec{i} + x\vec{j}$ around and across each of the following curves.
   (a) The circle $r(t) = \cos t\vec{i} + \sin t\vec{j},\ 0 \le t \le 2\pi$
   (b) The ellipse $r(t) = \cos t\vec{i} + 4\sin t\vec{j},\ 0 \le t \le 2\pi$

15. Find the flow of the velocity field $F = (x+y)\vec{i} - (x^2+y^2)\vec{j}$ along the upper half of the circle $x^2+y^2=1$ in the xy-plane.

16. Find the flux of the field F in the above problem(question 15) outward across the triangle with vertices $(1,0),(0,1),(-1,0)$.

17. Find the circulation of the field $F = y\vec{i} + (x+2y)\vec{j}$ around the square joining $(1,1),(-1,1),(-1,-1)$ and $(1,-1)$ counterclockwise.

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