Tutorial Sheet - 11

1. Sketch the region of integration and evaluate the following integrals:

   (a) $\displaystyle\int_0^\pi \int_0^x x\sin(y)\,dy\,dx$ (b) $\displaystyle\int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\,dx\,dy$

2. Find the centroid of the region in the first quadrant that is bounded above by the line $y = x$ and below by the parabola $y = x^2$.

3. Evaluate the following integrals:

   (a) $\displaystyle\int\int (x+y)^2 dx\,dy$ over the area bounded by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

   (b) $\displaystyle\int\int_S \sqrt{xy - y^2}\,dx\,dy$ where S is the triangle with vertices $(0,0), (10,1)$ and $(1,1)$.

4. Sketch the region of integration and evaluate $\displaystyle\int\int_R (y - 2x^2)\,dx\,dy$ where R is the region inside the square $|x| + |y| = 1$.

5. Evaluate $\displaystyle\int_0^{\pi/2} \int_0^{a\cos\theta} r\sqrt{a^2 - r^2}\,dr\,d\theta$.

6. Change the order of integration $\displaystyle\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V(x,y)\,dy\,dx$

7. Evaluate by changing the order of integration:

   (a) $\displaystyle\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ (b) $\displaystyle\int_0^\infty \int_x^\infty \dfrac{e^{-y}}{y}\,dy\,dx$ (c) $\displaystyle\int_0^1 \int_x^{\sqrt{2-x^2}} \dfrac{x}{\sqrt{x^2+y^2}}\,dy\,dx$

8. Find the smaller of the areas bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x + 3y = 6$.

9. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

10. Find the volume bounded by the xy plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$.

11. Find the volume of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

12. Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$.

$* * * * * * *$ END $* * * * * * *$