Indian Institute of Technology Ropar
Department of Mathematics

MA101 - Calculus
First Semester of Academic Year 2025-26

Tutorial Sheet - 10

1. In the following parts, find an equation for the plane, tangent to the level surface $f(x,y,z) = c$ at P_0. Also find the parametric equations for the line that is normal to the surface at P_0.
   (a) $x^2 - y - 5z = 0,\ P_0 : (2, -1, 1)$ (b) $x^2 + y^2 + z = 4,\ P_0 : (1, 1, 2)$

2. Find all the local maxima, local minima and saddle points of the following functions:
   (a) $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$.
   (b) $f(x,y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$.

3. Find the absolute maxima and minima of the functions on the given domain:
   (a) $f(x,y) = 2x^2 + y^2 - 4x - 4y + 1$ on the closed triangular plane bounded by the lines $x = 0, y = 2, y = 2x$ in the first quadrant.
   (b) $f(x,y) = x^2 - xy + y^2 + 1$ on the closed triangular plane bounded by the lines $x = 0, y = 4, y = x$ in the first quadrant.

4. Find the maximum value of $sin(A)sin(B)sin(C)$, if A, B, C are the angles of triangle.

5. Using the method of Lagrange Multipliers, find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $\dfrac{x^2}{8} + \dfrac{y^2}{2} = 1$.

6. Find the numbers a and b with $a \leq b$ such that $\displaystyle\int_a^b (24 - 2x - x^2)^{1/3} dx$ has its largest value.

7. Find the minimum distance from
   (a) the origin to a point on the plane $x + 3y - z = 6$.
   (b) the point $(1, 2, 0)$ to the cone $z^2 = x^2 + y^2$.

8. A rectangular box, open at the top, is to hold 256 cubic cm of sand. Find the dimensions for which the surface area (bottom and four sides) is minimized.

9. Assume that among all rectangular boxes with fixed surface area of 10 square meters there is a box of largest possible volume. Find its dimensions.

10. The Baraboo, Wisconsin, plant of International Widget Co. uses aluminum, iron, and magnesium to produce high-quality widgets. The quantity of widgets which may be produced using x tons of aluminum, y tons of iron and z tons of magnesium is $Q(x,y,z) = xyz$. The cost of raw materials is aluminum \$6 per ton, iron \$4 per ton and magnesium \$8 per ton. How many tons of each of aluminum, iron, and magnesium should be used to manufacture 1000 widgets at the lowest possible cost?

11. A firm uses wool and cotton fiber to produce cloth. The amount of cloth produced is given by $Q(x, y) = xy - x - y + 1$, where x is the number of pounds of wool, y is the number of pounds of cotton and $x > 1$ and $y > 1$. If wool cost p dollars per pound, cotton costs q dollars per pound, and the firm can spend B dollars on material, what should the mix of cotton and wool be to produce the most cloth?

12. Find the linear and quadratic approximations to $(3.98 - 1)^2/(5.97 - 3)^2$. Compare with the exact value.

13. Show that $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$ and find its linearization there. Then use it to approximate $f(1.1, -0.1)$.

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